Gegenstrebige Harmonik in the Music of Hans Zender

Robert Hasegawa, robert.hasegawa@mcgill.ca
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The German composer and conductor Hans Zender is a prominent figure in the world of contemporary European music. His music is played by orchestras and ensembles across Europe and, as a conductor, he has been a strong advocate for other composers, including Giacinto Scelsi and Helmut Lachenmann. Like many composers born in the 1930s, he was strongly influenced by the post-Webernian serialism of the 50s and 60s; later, though, he developed a strong interest in stylistic pluralism (inspired in part by one of his compositional heroes, Bernd-Alois Zimmermann), including a dialogue with music of the past, as evidenced in works like *Dialog mit Haydn* (1982), *Schuberts Winterreise: Eine komponierte Interpretation* (1993), and *Schumann-Fantasie* (1997). Other major influences on his music include Japanese music—*Muji no Kyo* (1975), *Lo-Shu I–VII* (1977–97)—and the poetry of Friedrich Hölderlin: *Hölderlin lesen I–IV* (1979–2001).

Since the late 1990s, Zender has been increasingly concerned with the possibility of combining the standard equal-tempered pitch gamut with the pure sounds of just intonation intervals (intervals with simple whole-number ratios between their frequencies). As Zender notes,

> For many music lovers it is a revelation to discover that intervals like the just major third or natural seventh have a luminosity, which compared to the color of a tempered interval is like a radiant red compared to a muddy red-brown.[1]

This revival of interest in pure intonation is inspired by the combination of a number of influences: Zender specifically mentions the role of the period performance movement in familiarizing listeners with just intonation, but one might also consider the work of American just intonation pioneers like Harry Partch and Ben Johnston, or the exploration of higher overtones by French spectralist composers.[2]

Zender used just ratios in works as early as the 1976 *Litanei* for three cellos, but they began to take a more central role in *Shir hashirim,* a 1996 setting of the Biblical Song of Songs.[3] According to Zender, the work's harmonic language is based on "the experience of the difference between equally tempered and 'natural' intervals of the spectrum."[4] *Shir hashirim* does not offer a synthesis of the two approaches to interval, but seeks rather to emphasize their uneasy coexistence. As Zender writes,

> The logically unsolvable problem of mediating between the overtone series and the equidistant scale, between the perception of qualities and linear, quantitative thinking appears in the harmony of *Shir hashirim* as a paradox: a *gegenstrebige Harmonik* (a harmony of opposing tensions).[5]

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[1] Hans Zender, "Gegenstrebige Harmonik" in *Musik-Konzepte: Musik der anderen Tradition: mikrotonale Tonwelten* (Munich: Edition Text + Kritik, 2003), 167–208: 207. Translation of this and all other excerpts by the author.

[2] "Gegenstrebige Harmonik," 169.

[3] Earlier Zender works employing microtonal resources also include *Kantate nach Meister Eckhart* (1980) and *Dialog mit Haydn* (1982).

[4] Hans Zender, "Wegekarte für Orpheus? Über nichtlineare Codes der Musik beim Absteig in ihre Unterwelt," in *Die Sinne denken: Texte zur Musik 1975–2003,* ed. Jörn Peter Hiekel (Wiesbaden: Breitkopf & Härtel, 2004), 85–94: 91.

[5] "Wegekarte für Orpheus?" 94. In *Shir Hashirim,* Zender applies frequency-based virtual ring modulation techniques (to be discussed later in this paper) to equal-tempered parent tones *(Elterntöne),* emphasizing this disjunction between the two types of interval.

Zender expanded upon the theoretical underpinnings of this new approach in his 2000 lecture "Gegenstrebige Harmonik," delivered at the Darmstadt International Summer Courses for New Music. The concept of *gegenstrebige Harmonik* is inspired by Heraclitus's Fragment 51: "They do not understand how that which differs with itself is in agreement: harmony consists of opposing tensions, like that of the bow and the lyre."[6] The physical harmony of the bow and lyre is based on the tension inherent in their construction. The bent bow pulls the bowstring in opposite directions, keeping it taut; similarly, the strings of the lyre are kept tight, allowing them to sound. Heraclitus underlines the way that contrary forces, rather than destroying harmony by their conflict, can instead produce harmony when they balance one another.

In *Shir hashirim,* as in later works, these "opposing tensions" arise from two fundamentally different ways of conceiving intervals between pitches. One is the equal-tempered, quantitative world of symmetrical, equal octave subdivisions; the other is the just-intonation, qualitative world of frequency ratios. Thinking in terms of temperament is useful in conceptualizing abstract geometries (epitomized by twelve-tone music or pitch-class set theory), while thinking in terms of frequency draws our attention to an interval's sonic quality: for example, the specific acoustical consonance of a just 5:4 major third. Zender writes:

> In principle, no mediation is possible between these two manifestations of the interval; hearing pure (perfect) intervals is an experience of quality, while hearing tempered intervals represents the experience of a quantity-governed tone order generated by culture.[7]

One might contrast these two ways of thinking about intervals by comparing the way each approach would subdivide the interval of the octave (the common reference interval between the two approaches). Thinking in terms of equal temperament, one would divide the octave into two identical equal-tempered tritones, but thinking in terms of frequency ratios one would instead subdivide the octave into a just perfect fifth (3:2) below a perfect fourth (4:3). Composers of the "spectral school," including Tristan Murail and Joshua Fineberg, have also pointed out the opposition between these two approaches to pitch organization, and use the term "frequential" to refer to pitch structures conceived in terms of frequency instead of conventional tempered pitch space.[8]

As Zender freely admits, the idea of *gegenstrebige Harmonik* is not really a new concept at all: as he points out, the tension between these opposite ways of conceiving interval has always been essential to musical practice. Since the advent of polyphony, Western music has explored the contrast between a vertical organization of sounds according to certain consonances, and a linear organization driven by melodic shape. Zender characterizes music history as, at least in part, the result of a constantly evolving tension between the vertical and the horizontal.

> For a thousand years, musical thinking in the framework of European culture has been occupied with the development of the ability to apprehend sounding shapes as multidimensional, to perceive composite events out of simultaneously occurring component parts. In the course of historical development, the two extreme poles of this opposition (the simultaneous sounding of multiple pitches and the superposition of

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[6] Heraclitus, Fragment 51, translated by Kathleen Freeman, *Ancilla to the Pre-Socratic Philosophers* (Oxford: Oxford University Press, 1948), 28. In the closely related Fragment 8, Heraclitus states, "That which is in opposition is in concert, and from things that differ comes the most beautiful harmony" (op. cit., 25).

[7] Hans Zender, "A Roadmap for Orpheus?" in *Theory into Practice: Composition, Performance, and the Listening Experience,* ed. Peter Dejans (Leuven: Katholieke Universiteit Leuven, 1999), 108–116: 111–112.

[8] Tristan Murail, "Villeneuve-lès-Avignon Conferences, Centre Acanthes, 9–11 and 13 July 1992," trans. Aaron Berkowitz and Joshua Fineberg. *Contemporary Music Review* 24, nos. 2–3 (2005): 196.

> different linear structures) contrast with one another; they are articulated differently in changing style periods and confront one another. Simultaneities appear in principle—even if they are so-called dissonances—as the unfolding of fundamental tones: they have a spectral character. The superposition of lines is oriented around these basic sounds: the unique dynamic of the line brings these only for short moments into an opposition with the harmony.[9]

In addition to exploring this fundamental contrast between two conceptions of pitch, another of Zender's reasons for turning toward just-intonation intervals is a dissatisfaction with the pitch resources of the equal-tempered twelve-tone scale:

> A growing number of composers have problems with our conventional tempered tone system, because they no longer know what these intervals "mean." Expressed another way: these composers have developed a very highly differentiated concept of interval in their "inner ear"; they distinguish between thirds, fourths, seconds, etc. of diverse order—and the averaged-out values of the chromatic system … give them no possibility of signifying these differences. On the contrary, the tempered intervals seem to have laid themselves like a fog over the intervallic sensitivity of most musicians. […] The qualitative differences between intervals present in the consciousness of older cultures are in danger of being leveled into a diffuse chromatic soup.[10]

The brilliant and contrasting colors of just intonation allow a far greater differentiation of interval quality than is possible in the tempered twelve-tone system. The twelve-tone tempered scale was originally designed to allow the approximation of just intonation triads in a variety of keys: purity of intonation was sacrificed to allow a greater range of modulation and the conceptual simplicity of an equidistant chromatic scale.

As Zender argues, the dramatic changes in harmony of the early twentieth century sever the equally tempered scale from its historical and thus acoustical origins. Temperament means the loss of the historical "meanings" of intervals, since the same played interval can be taken to "mean" several different just intonation ratios. In the twentieth century, harmony is "for the first time no longer thought of as based on a fundamental tone (*Grundton*), but rather as the simultaneous appearance of intersecting lines."[11] This characterization is especially applicable to serial music, which to Zender represents a triumph of the linear and quantitative over the vertical, qualitative experience of interval. The devaluation of the qualitative aspect of hearing collapses the *gegenstrebige* nature of previous harmonic practice, which balanced rationality and sensuality, in favor of an entirely quantitative, linear mode of thought. In Zender's view, this collapse creates a fundamental "problem of pitch" for contemporary composers, an exhaustion of pitch resources epitomized by Ligeti's cluster works from the 1960s. Zender goes on to discuss how this "problem of pitch" has been explored in recent work by Helmut Lachenmann and Brian Ferneyhough.

Lachenmann's cultivation of a musique concrète instrumentale seems to offer one way of dealing with this problem; Zender characterizes Lachenmann's approach as a manifestation of the motto "down with pitch—onward to noise." The scarcity of clearly defined pitches in Lachenmann's maelstroms of noise lend them a certain rarity value: "Through the aesthetic valuation of noise, pitches again become something strange; they make themselves rare and can thus in part regain some of their

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9 "Gegenstrebige Harmonik," 167.
10 "Gegenstrebige Harmonik," 168–169.
11 "Gegenstrebige Harmonik," 167.

lost efficacy." This maneuver, however, doesn't confront the "problem of pitch" so much as avoid it: in Lachenmann's music, the problem is "deferred but not solved."[12]

Zender characterizes the music of Ferneyhough as a further extension of the excessive linearity of serial music, separating the vertical and horizontal dimensions to a still greater degree. Ferneyhough's layered processes of pitch and rhythmic derivation create a music that "swings between extreme rationality in the horizontal and extreme irrationality in the vertical construction." As a result, the experience of interval as acoustical quality is overwhelmed by the intellectual demands of the music's linear structures: Zender warns that this can produce "an atrophy of sensory perception." As a result, the ideal balance of the vertical and horizontal, the sensual and the intellectual, is lost—one can no longer apprehend a structure that is meaningful both in the moment and diachronically, as in successful works of the Western tradition. To Zender, this situation mirrors the state of the highly technological modern world, which presents "a surplus of information and an individual, who can no longer see this information as a whole."[13]

As this discussion of Lachenmann and Ferneyhough demonstrates, Zender's idea of the "harmony of opposed tensions" between the qualitative and quantitative concepts of interval also expresses itself in a constellation of related ideas. He identifies a similar tension between the vertical and horizontal dimensions of music, between two different ways of experiencing time (the instantaneous and sensual versus the reflective and rational), and between traditional societies and today's scientific, technological world.[14] Though Zender occasionally seems poised to decry modern musical practice in favor of a return to tradition and "quality-based" intervals, his position is actually more complex and nuanced, recognizing the impossibility of "going back" to a prelapsarian state, and the need to establish a compromise between the two ways of thinking about interval. It makes no sense, he argues, to insist on a tempered, "purely chromatic" system that would suppress the acoustical qualities of interval—and it would be equally senseless to abjure temperament in favor of ratio-based just intonation (as in some of the writings of Ben Johnston). Tempered thought is "simply the opposition to thinking in whole-number ratio intervals: both thought forms have their history, their interferences, their synergies: they remain dependent on one another."[15]

Zender's concept of the relationship between frequency-based and tempered thought is reflected in his pragmatic approach to pitch notation. His notation switches between several different levels of pitch specificity, but the underlying harmonic conception doesn't change when he moves from one level of specificity to another—rather, the same underlying harmony is represented with varying

[12] "Gegenstrebige Harmonik," 170–171.
[13] "Gegenstrebige Harmonik," 171. Zender is emphatic that these remarks are in no sense intended as a polemic against Lachenmann or Ferneyhough—in fact, Zender is a strong advocate of Lachenmann's music, and has recorded a number of his works as a conductor—but rather represent an attempt to situate his own approach to the "problem of pitch" within the contexts of other strands of contemporary musical thought.
[14] Zender's contrast between two ways of conceiving musical time will have multiple resonances for American theorists. Like Zender, Jonathan Kramer draws a distinction between "vertical" and "linear" ways of experiencing musical time in his book *The Time of Music* (New York: Schirmer, 1998). In a recent article, Daniel Harrison contrasts a Heraclitean mode of listening, attentive to minute fluctuations of sound in all of its sensual detail, with an "arch-Platonist" mode of listening that understands musical events as exemplars of more abstract categories detached "from the particularity of actual sound." Harrison warns of the dangers of pursuing either of these modes to the polar extremes, and notes that music theorists have tended to fall much more on the "Platonic" side of the divide. ("Non-Conformist Notions of Nineteenth-Century Enharmonicism," *Music Analysis* 21, no. 2 (2002), 115–160.) One might also consider Carolyn Abbate's discussion of drastic versus gnostic listening, which separates structural, formalist thinking ("gnostic") from the embodied, moment-to-moment experience of music ("drastic"). ("Music: Drastic or Gnostic?" *Critical Inquiry* 30, no. 3 (2004), 505–536.)
[15] "Gegenstrebige Harmonik," 207.

degrees of precision. (This is comparable to the practice of spectralist composers like Murail or Gérard Grisey, who round off their frequency-based spectra to the nearest quarter- or eighth-tone.) In some cases, Zender does not specify microtones at all, and frequencies that fall "between the cracks" of twelve-tone equal temperament are simply rounded to the nearest semitone. In other cases, Zender uses arrows attached to accidentals to indicate quartertone inflections, yielding twenty-four tones per octave. At its most detailed, Zender's notation differentiates pitches a sixth of a semitone (a twelfth-tone, or 16 2/3 cents) apart—the symbols + and - in parentheses above a note modify its pitch up or down by a sixth of a semitone, and can be combined with one another as ++ and -- to change the pitch by a third of a semitone. The use of twelfth-tones, which divide the octave into seventy-two distinct pitches, has also been a feature of music by composers including Franz Richter Herf, Joe Maneri, Ezra Sims, and James Tenney. The fine-grained seventy-two-tone equal temperament allows very close approximations of ratio intervals including the 7:4 natural seventh (969 cents) and the just major (5:4) and minor (6:5) thirds (386 and 316 cents). Zender's flexible level of pitch specificity allows him to choose when to insist on very exact intonation, and when to settle for a looser approximation. His experience as an orchestral conductor seems to have given him a clear sense of what demands are reasonable for instrumentalists: instruments that are less accustomed to playing microtones (trumpet and oboe, for instance) rarely have microtonal inflections in their part, and fast-moving lines are typically written without microtonal details.

In "Gegenstrebige Harmonik," Zender sets a number of goals for a new approach to compositional theory. The theory must "make the finest microtonal motions hearable and understandable" while also accommodating the "historical genesis of our concept of interval" (the simple ratios of just intervals). The theory should be "founded on a wholly new network of relationships, equally far from the certainties of the old tonality as from those of serially ordered atonality, a network, that nevertheless also can represent these and still other states of historical thought."[16] Zender's essay includes a number of musical examples, but most of these are contrived for the purpose of demonstrating specific theoretical ideas, and do not indicate how Zender applies his theoretical ideas in actual works. In this paper, I will illustrate the musical results of Zender's harmonic theories with original analyses of excerpts from two recent works: *Music to Hear,* a 1999 setting of Shakespeare's eighth sonnet for soprano and chamber ensemble, and *Bardo* (written in 2000), a concertante work for solo cello and chamber orchestra inspired by the Bardo Thodol, or Tibetan Book of the Dead. Zender's recent music shows an unusual degree of transparency for the analyst—an analyst with a good understanding of overtones and just intonation can often reconstruct the chain of processes used to compose the music. The theoretical underpinning of Zender's work lends it a unique kind of musical logic, allowing subtle quasi-tonal modulations and the construction of pitch "genealogies" reminiscent of David Lewin's transformation networks.

Theoretical and analytical implications

For the analyst approaching Zender's music, one of its most striking characteristics is the way that the same pitch can take on very different musical senses depending on its position in relationship to the fundamental of an overtone-based harmony.[17] For reference, Example 1 shows the first 32 partials of the overtone series on C. (This example adopts Zender's microtonal notation, based on tempered twelfth-tones, which divide each semitone into six parts.) If middle C is heard as the eighth

[16] "Gegenstrebige Harmonik," 173.

[17] Zender's clearest illustration of this idea appears as a "Modulationen-Kette" in his Example 7 ("Gegenstrebige Harmonik," 187), a long chain of modulations based on reinterpretation of a single pitch. A comparable set of reinterpretions occurs in Example 17a ("Gegenstrebige Harmonik," 205).

partial of a fundamental on C_0, it will have a different musical character than if heard (for example) as the third partial of F_2. This is closely related to Hugo Riemann's idea of "tone representation," as described in his 1915 article "Ideen zu einer 'Lehre von den Tonvorstellungen'": "According to whether a note is imagined as 1, 3, or 5 of a major chord or as I, III, or V of a minor chord, it is something essentially different and has an entirely different expressive value."[18]

The tone representation of a given pitch or set of pitches as partials above a fundamental frequency is highly dependent on musical context. The analytical method applied here compares the input set of pitches to a template based on the harmonic series, and assigns the fundamental frequency whose harmonic series produces the closest intonational fit and the simplest interval ratios between pitches of the set.[19] A strength of this method of analysis is its high sensitivity to the registral positions of pitches—unlike pitch-class set theory and related models, it does not generalize pitches into pitch-classes. Zender is sharply critical of pitch-class set theorists' inattention to register, which to him is essential for the harmonic sense of any structure.[20]

The changing tone representation of a pitch can be heard clearly in a passage from the fifth and final tutti section of Zender's *Bardo:* here, the D above middle C is heard in a number of different harmonic contexts. Example 2 shows my analysis of the basic harmonic progression: changing fundamentals are designated by letter names, and each pitch is numbered according to its overtone relationship to that fundamental. For example, in the first chord D is the fourth overtone of the fundamental, E is the ninth, the lowered B is the thirteenth, and the E♭ is the seventeenth. Throughout the passage, chords appear in pairs sharing a similar selection of overtones: with a few exceptions, D4 is the top or bottom note of each chord. This figure omits the music between each pair of chords—usually this consists of glissandi in the solo cello or the timpani.

This kind of "modulation" can apply to dyads (or even larger sets) as well as individual pitches. In Example 2, we can observe instances where the same notated interval appears in different harmonic contexts, representing different members of the overtone series. The D and B♭ of the seventh chord are understood as overtones 5 and 8 of a B♭ fundamental; the same D and B♭ in the tenth chord are overtones 9 and 14 of a C fundamental. In a pure tuning, these intervals would have different sizes: the 5:8 minor sixth of chord 7 is about 814 cents, while the 9:14 interval of the tenth chord is only 765 cents, about a quarter tone flatter. (For another example, compare the two representations of the D–G fourth in chords 3 and 12—the first is 3:4, or 498 cents, while the second is 14:19, or 529 cents.) Only after temperament rounds off each interval to the nearest semitone can the two intervals be regarded as equivalent; "modulation" of this type, where an interval pivots between two different acoustical meanings, is only possible through "tempered thinking." For Zender, there are pros and cons to adopting this mode of thought: tempered thinking results in the pollution of the "acoustical environment" of just intervals with impure approximations, but also allows a new possibility, "the

[18] Hugo Riemann, "Ideas for a Study 'On the Imagination of Tone'," trans. Robert Wason and Elizabeth West Marvin, *Journal of Music Theory* 36, no. 1 (1992): 86.

[19] A more detailed consideration of the theoretical implications of this approach appears in Robert Hasegawa, "Gérard Grisey and the 'Nature' of Harmony." *Music Analysis* 28, no. 2–3 (2009), 349–370. A related approach is outlined in Manfred Stahnke, "Zwei Blumen der reinen Stimmung im 20. Jahrhundert: Harry Partch und Gérard Grisey." *Hamburger Jahrbuch für Musikwissenschaft* 17 (2000): 369–89, esp. 386–387.

[20] "Gegenstrebige Harmonik," 175–176.

capability to instantly reinterpret a sounding pitch's harmonic meaning, in fact even to recognize a double or multivalent meaning for the same sound."[21]

Harmonic context is essential to our perception of the two D–B♭ intervals as different—when played in equal temperament, they are sonically identical. The composer can influence which ratio the listener perceives by changing the harmonic surroundings—a particularly effective device, Zender notes, is simultaneously to play pitches corresponding to the sum and difference tones of a given ratio's ideal frequencies.[22] Very small microtonal differences in the size of an interval are exaggerated and made audible by the addition of the sum and difference tones. First described by Tartini and Sorge in the eighteenth century, combination tones are a subjective effect produced by a non-linear response of the auditory system. Two generating frequencies (A and B) can produce several different types of combination tones, including the simple difference tone B - A, the cubic difference tone 2A - B, and the summation tone A + B. The combination tones generated by equal-tempered intervals between a minor second and a double octave are displayed in the chart in Example 3. The resulting sum and difference tones will always belong to the same overtone series as the starting interval. Sum and difference tones are usually only perceived in rare acoustic situations,[23] but in Zender's music the sum and difference frequencies are actually performed as part of the sounding harmony. As Zender writes, "The clarity of the intended interval ratio arises not from more exact intonation, but from the composer's harmonic technique: through the composing-out of the sum and the difference of the interval, the ear receives the information necessary to decipher the interval's ratio."[24] These summation tone harmonies are the basis of Zender's harmonic language: returning to Example 2, one can note the way that all the upper members of each chord can be understood as sums (or sums of sums) of the bottom two overtone numbers.

Example 4 (based on Zender's own Example 15) shows seven tritones, each with a different frequency ratio. The sizes of the tritones are shown in cents. By playing the sum and difference tones simultaneously with the tritone, the intended interval ratio can be expressed clearly. For example, playing a G below the tritone C–-F¼sharp interprets the tritone as 8:11 (G is 11 - 8, or 3). A slightly raised A♭ in the bass will reinterpret the tritone as 5:7, while an A shifts the perceived ratio to 12:17. Note that due to the nature of the overtone series, the difference tone A - B varies much more than the sum tone A + B—for this reason, the difference tone tends to play a stronger role in determining the harmonic meaning of the pitches by which it is generated.

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21 "Gegenstrebige Harmonik," 186.

22 "For the last ten to fifteen years, I've been preoccupied with microtonality as a compositional problem. In the course of time, I developed my own technique when dealing with harmony, which is defined like this: you start with a basic interval, which comes from our normal, tempered system—a fourth, a third, or whatever. And then you work out the sums and differences, i.e. the combination tones, and add them to that basic interval. This is quite a natural process, because we can hear those combination tones at a very low volume when we play and sing. However, usually, their volume is so low that we just aren't conscious of them. My technique incorporates those additional tones into my compositions—and I even put them down on paper. Thus I am forced to think in a microtonal way, because those intervals usually don't fit our tempered system." Interview with Hans Zender, *Giacinto Scelsi/Hans Zender: Klang und Sinn.* Film by Peider A. Defilla, 2007. Wergo-NZ64.

23 As demonstrated by the early-twentieth-century psychologist Edward B. Titchener, even though combination tones are typically difficult to perceive, a listener can train him- or herself to hear them more clearly. See Eric Schwitzgebel, "Introspective Training Apprehensively Defended: Reflections on Titchener's Lab Manual." *Journal of Consciousness Studies* 11, nos. 7–8 (2004), 58–76.

24 "Gegenstrebige Harmonik," 207.

Harmonies based on sum and difference tones have been of great interest to a number of composers, including Grisey, Murail, Sims, and Claude Vivier. Vivier used chains of summation tones to expand dyads into large orchestral chords that he called "les couleurs."[25] In the music of Grisey and Murail, summation and difference tones often appear as a realization of models based on the electronic studio effects of ring modulation and frequency modulation, each of which produces new frequencies based on the addition and subtraction of input frequencies from a carrier frequency. Zender refers specifically to the inspiration of Stockhausen's *Mantra* (1970), in which piano sounds are ring-modulated with tones from a sine-wave generator. Zender describes the effect of the ring-modulator in the piece: it "works like an exotic color, which brightens the gray of the chromatic basic materials. The effect is fascinating—perhaps it is a fascination through bafflement, through the bafflement of an unresolved contradiction." This is another example of Zender's *gegenstrebige Harmonik*: a "harmony of opposed tensions" between the equal-tempered sound-world of the piano and the frequency-based, microtonal ring-modulated sounds. For Zender the ring modulator not only colors the input interval, but also "interprets" it: by adding the sum and difference tones, the modulator redefines a harmonically ambiguous input interval as a specific ratio between harmonic partials, in reference to an assumed fundamental frequency.

> The ring modulator adds to every interval it receives (whether in just intonation or not) the difference and the sum of the frequencies, and makes these audible. When it does so, it interprets the input interval as an excerpt of an imaginary harmonic spectrum—if the interval has a very complex ratio, the fundamental frequency of this spectrum may lie at a pitch below the threshold of human hearing. The sum and difference tones are partials of this same spectrum.[26]

Adding sum and difference tones to an interval is just one of several ways to guide the listener toward interpreting it as a specific just interval ratio. The most powerful is probably the sounding of the fundamental itself under an interval; other low partials of the same harmonic series can have a similar effect. These aids to interval recognition (or *Hilfsvorstellungen,* to use a Riemannian term) can also occur above the interval in question—upper partials of an overtone series can also serve to orient the ear. As Zender notes, without a coherent harmonic context even the most carefully tuned just intervals can sound like mere smudgings of familiar tempered pitches:

> [The microtonal composer] must do everything possible to ensure that his forms reach the ear with optimal clarity, helping the ear, so to speak, to find the path. What use are quarter-, eighth-, and twelfth-tones if the ear is not guided to interpret them correctly? They will be heard as only smearings, nothing but variations on the conventional.[27]

For composers interested in extending their pitch material into microtones, the difficulty lies not in finding more pitches—after all, the continuum of pitch is infinitely subdivisible—but rather in finding more meanings for tones, more ways that they can enter into significant musical relationships. If these meanings are lacking, the microtones will tend to be heard as mere "out-of-tune" inflections of standard pitch materials.

We have already seen ring modulation techniques in Example 2; another instance can be observed in the excerpt from Zender's 1999 *Music to Hear* analyzed in Example 5. This excerpt sets the second line of Shakespeare's eighth sonnet, printed in full as Example 6. In Example 5, diamond-shaped noteheads indicate the generating interval for each chord—these are expanded into four-note

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[25] Gilmore, Bob. "On Claude Vivier's 'Lonely Child'." Tempo no. 239 (2007), 2–17.

[26] "Gegenstrebige Harmonik," 177.

[27] "Gegenstrebige Harmonik," 173.

harmonies by adding the sum and difference of each of the frequencies of the generating interval. Zender describes how, through this process, each simple interval is changed into a kind of “cube”; he suggests that this could be thought of as a kind of “cubist harmony.”[28] Note how Zender’s technique creates a wide variety of verticalities with different implied “roots” (shown by letter names below the staff) that nonetheless share a common generating procedure, and arguably a general sonic similarity due to their origin as “ring-modulated” sonorities.

The harmonic ring modulation of Example 5 is the simplest version of the technique; in other contexts, Zender uses related procedures to unfold complex networks of related pitches. I’ve termed these networks genealogies, since they closely resemble the parent–child relationships of a family tree. Each node of the genealogy represents the frequency of a pitch, expressed as an overtone of a notional fundamental (which may or may not be actually heard). Any pair of nodes can be “parents” to a third node, which is the sum or difference of their frequencies. Zender himself does not explicitly invoke the idea of genealogy, but he does often use the term *“Elterntöne”* (parent tones) to describe the generating pair of pitches.

Example 7 presents an analysis of the opening cello solo of *Bardo,* illustrating how a genealogy of combination tones unfolds in the excerpt. (Note that the solo part is written for a custom curved bow that allows the cellist to play all four strings simultaneously.) In this annotated score, letter names indicate the pitch classes of the implied fundamental frequencies—the numbers show how each pitch is interpreted as a harmonic partial of its fundamental, and also its derivation as a sum or difference tone of previously heard notes. Occasionally, a single pitch is reinterpreted as a partial of a different fundamental—these “pivot modulations” are shown on the score by horizontal arrows.

Example 8 extracts from Example 7 a genealogy of pitch relationships: the curved lines between the numbers show how two “parent” tones give rise to a “child” that is the sum or difference of their frequencies. All the pitches at the beginning of the excerpt are derived from the initial whole tone C#–D#, the eighth and ninth harmonics of a notional fundamental on $C\#_{-1}$. The integers 8 and 9 give rise to two arithmetic sequences. The first is produced by repeatedly adding 8 to 9: 17, 25, 33, 41. The second is produced by repeatedly adding 9 to 8: 17, 26, 35, 44, 53, 62. As a result, each pitch is the summation tone of either 8 or 9 with another sounding pitch.

Music theorists will recognize a similarity between this type of genealogy and the transformational networks developed by David Lewin. Like Lewin’s networks, genealogies are made up of nodes and arrows; changing the contents of the input nodes of a genealogy would give rise to a different, but closely related network. A basic feature, however, lends genealogies a rather different character: while Lewin’s networks chart the transformation of single entities by functions, genealogies track how two entities give rise to a third. Genealogies could thus be considered an expansion on Lewin’s networks, along the lines suggested by Julian Hook, who writes:

> Even the basic idea that transformations must be functions—specifically, functions of a single variable—might be called into question. Such a function is a special kind of relation between two objects; relations that are not functions are surely useful, as are relations involving more than two objects. […] The challenge here is to increase flexibility as needed without unduly increasing complexity, decreasing usability, or—one would hope—doing too much violence to the formal elegance of Lewin’s theory.[29]

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[28] “Gegenstrebige Harmonik,” 188.

[29] Julian Hook, "David Lewin and the Complexity of the Beautiful." *Intégral* 21 (2007): 186.

With the metaphor of genealogy in mind, one can understand Zender's attraction to Shakespeare's eighth sonnet, with its comparison of musical harmony to domestic order. Like *Bardo,* Zender's *Music to Hear* often presents two "parent" tones giving rise to one or more combination tones, which can then produce "children" of their own. Example 9 analyzes Zender's setting of the ninth line of the sonnet, "Mark, how one string, sweet husband to another …" In the first line of this example, the octave B♭s and the subsequent D♭ (overtones 5, 10, and 12 of a G♭ fundamental) are the "parents" of all the musical material. The words "one string" are illustrated musically by a single sustained D♭ on the cello. After the voice completes the line, the strings respond with a distorted echo, all based on summation tones built from the violin's held A♭. Example 10 is a genealogy of the pitches in this excerpt. Note that the last seven chords share the same graph, but different input notes result in different pitch formations.

Zender frequently chains together combination tones to produce successions of pitches analogous to the Fibonacci series, where each new frequency is the sum of the two preceding frequencies.[30] As shown in Example 11, no matter what the starting pitches, all Fibonacci-type series converge to a single repeating interval size of approximately 833 cents—this is the frequency ratio of the "Golden Mean," approximately 1.618:1. The example shows three different Fibonacci-type series with different starting intervals. Whatever the starting interval, the intervals between successive notes quickly settle near 833 cents—an unending series of slightly stretched minor sixths.

Example 12 illustrates Zender's use of this technique in Bardo, as the cello soloist begins the transition into the first tutti section. The excerpt begins with three-note gestures in which the last note is the sum of the previous two. In measure 37, the C and C# start a Fibonacci-type chain of summation tones: 16, 17, 33, 50, 83, 133, 216, and 349. At measure 41, Zender subposes a low C, which recontextualizes the high F 1/4# as a 22nd overtone (very close in terms of pitch class to the 349th overtone). The cello gradually wedges outward from the F-1/4#, eventually filling in all the overtones from 16 to 26.

In these examples, we've seen how Zender brings together the "opposing tensions" of tempered pitch geometries and just intonation frequency ratios. By recognizing the unresolved (and perhaps irresolvable) tensions within the most basic musical concept—the interval—Zender arrives at a musical system with room for both intervallic conceptions: the acoustical purity and overtone implications of the just intervals are combined with the modulatory flexibility and notational simplicity of a fine-grained equal temperament. Sum and difference tones not only help to stabilize and clarify harmonic intervals, but also give rise to unique new networks of pitch relationships, unfolding coherent genealogies that link each new pitch to its forebears. Zender concludes his essay by observing that the idea of a "harmony of opposing tensions" might at first seem a logical contradiction or paradox—but that for the composer, these tensions provide a fertile environment from which artistic creation can emerge.[31]

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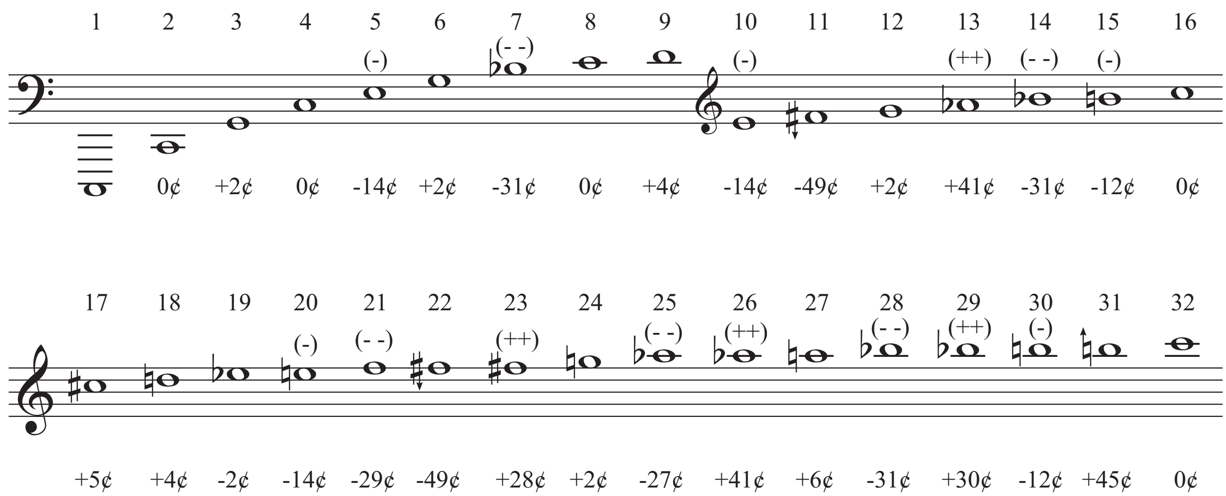
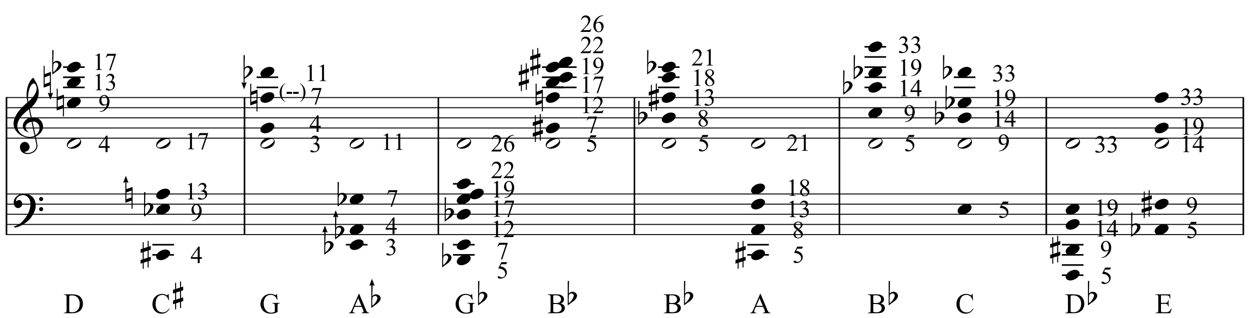
30 "Gegenstrebige Harmonik," 195–196.

31 "Gegenstrebige Harmonik," 208.

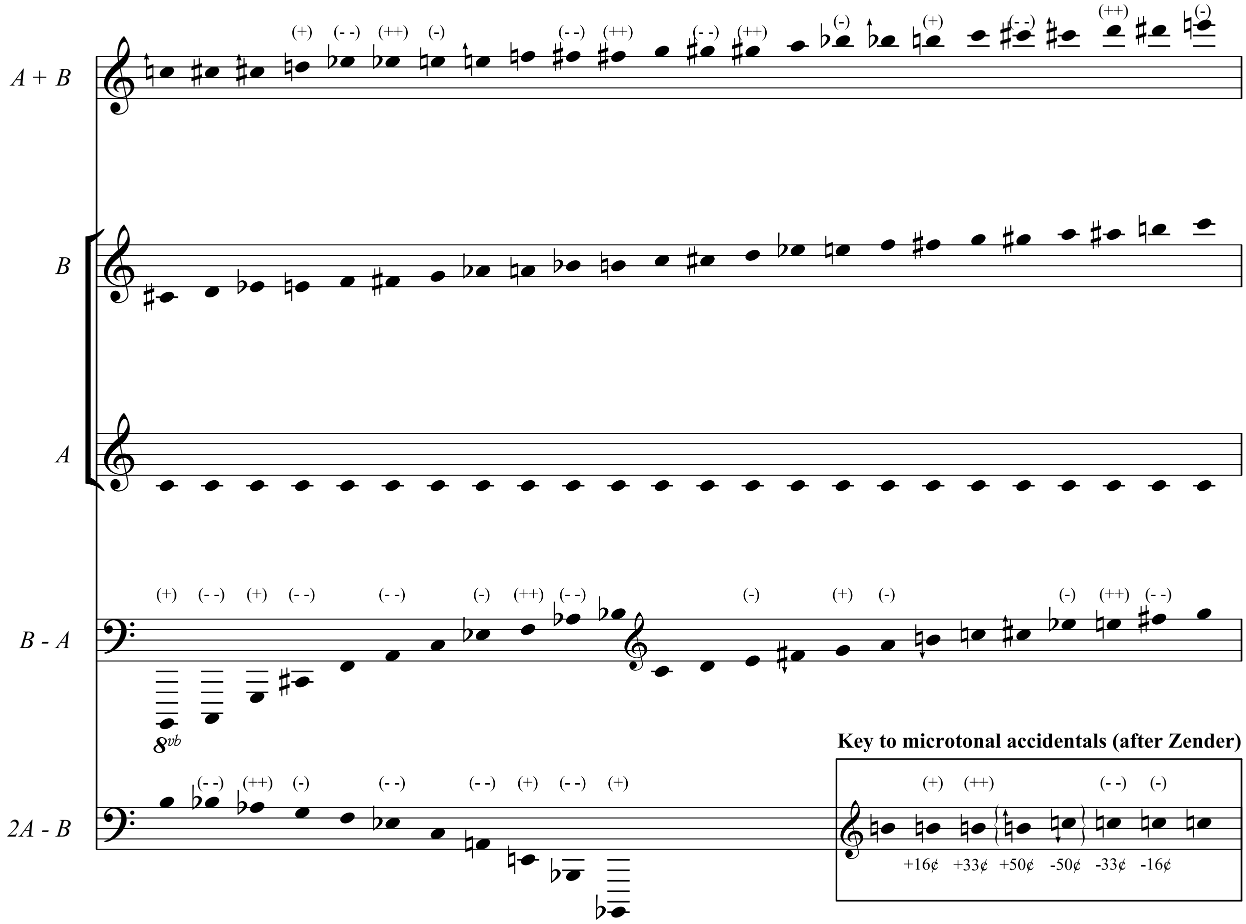
Examples



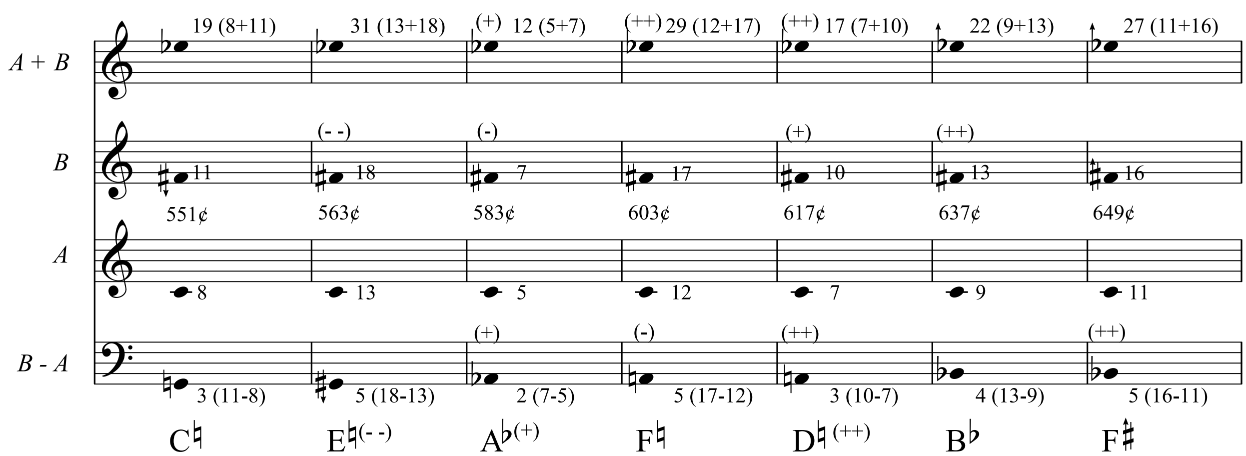
EXAMPLE 1: Overtone series with cent deviations from equal temperament
Hans Zender's microtonal notation is based on division of the semitone into six equal parts. Raising or lowering a note by one sixth of a semitone (16 2/3 cents) is designated by a parenthesized + or - above the note. Alterations of one-third of semitone are shown by ++ or --, and alterations of half a semitone (a quarter-tone) by arrows attached to the accidental. Based on Zender's "Gegenstrebige Harmonik," in *Musik-Konzepte: Musik der anderen Tradition: mikrotonale Tonwelten* (Munich: Edition Text + Kritik, 2003), 167–208: 188.



EXAMPLE 2: Changing "tone representations" in *Bardo* (2000), mm. 376–394
Pitch-class letter names indicate fundamentals; numbers designate the position of each pitch in the overtone series of that fundamental. Barlines separate pairs of related chords. Omitted in this example are the glissandi passages for timpani or solo cello that separate each chord pair.

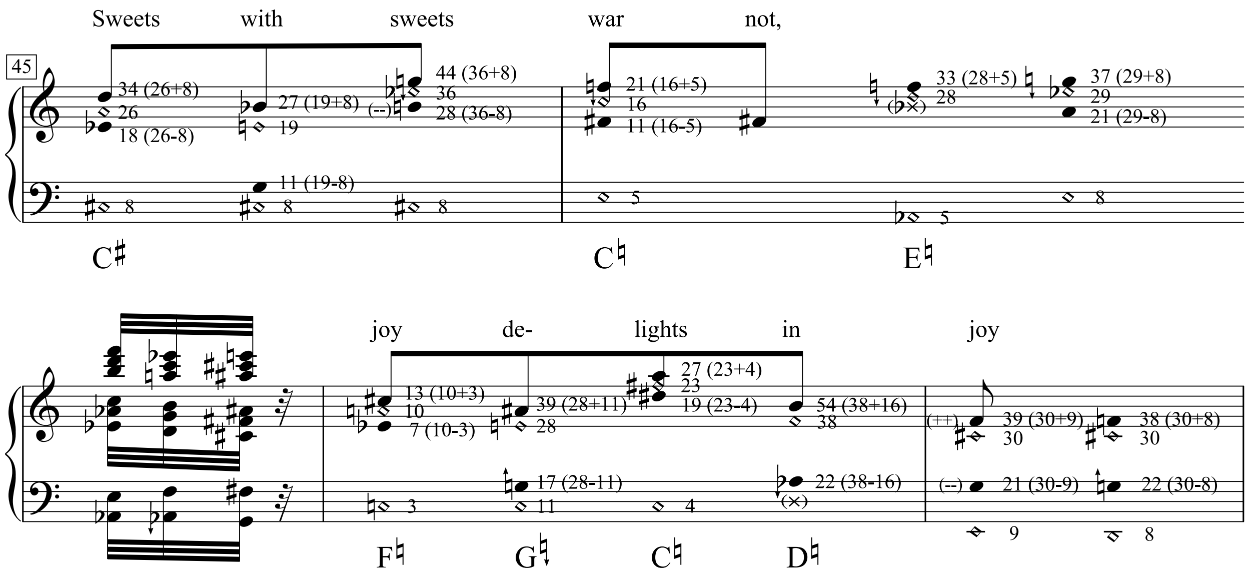


EXAMPLE 3: Sum and difference tones of equal-tempered intervals from 1 to 24 semitones
This chart shows the sum and difference tones generated by the pair *A* and *B*. In addition to the simple sums and differences, the chart includes the cubic difference tone *2A - B*, also a common psychoacoustical phenomenon.



EXAMPLE 4: Tritones
Interval ratios for seven “tritones” of different sizes, shown in cents between the middle two staves. Zender observes that playing the sum and difference tones simultaneously with the generating interval

influences how a listener interprets the tritone as an interval ratio. Implied fundamentals are indicated by letter names. Based on “Gegenstrebige Harmonik,” 203.

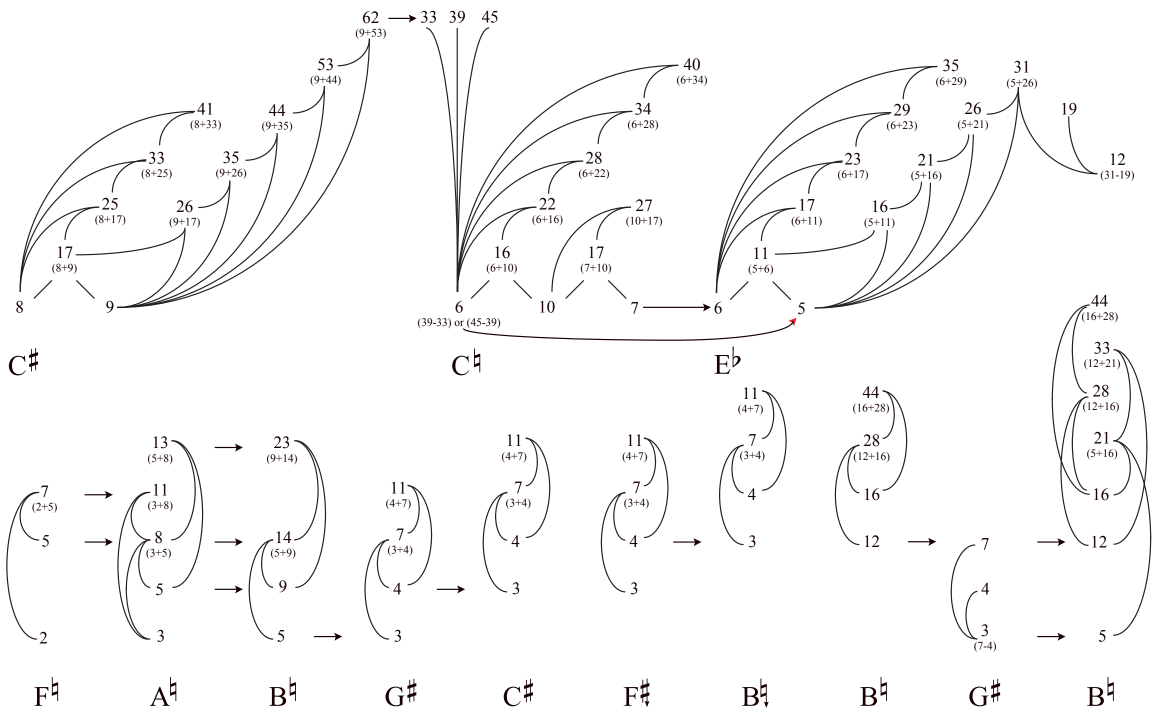


EXAMPLE 5: “Ring modulation” harmonies in *Music to Hear* (1999), mm. 42–49
As in Example 2, letter names and numbers are used to describe each pitch as an overtone of a fundamental. Diamond-shaped noteheads show the generating pitches for each chord. x-shaped noteheads indicate pitches that should theoretically be present, but are omitted in the score.

1. Music to hear, why hear’st thou music sadly?
2. Sweets with sweets war not, joy delights in joy:
3. Why lov’st thou that which thou receiv’st not gladly,
4. Or else receiv’st with pleasure thine annoy?
5. If the true concord of well-tuned sounds,
6. By unions married, do offend thine ear,
7. They do but sweetly chide thee, who confounds
8. In singleness the parts that thou shouldst bear.
9. Mark how one string, sweet husband to another,
10. Strikes each in each by mutual ordering;
11. Resembling sire and child and happy mother,
12. Who, all in one, one pleasing note do sing:
13. Whose speechless song being many, seeming one,
14. Sings this to thee: “Thou single wilt prove none.”

EXAMPLE 6: Shakespeare’s Sonnet No. 8
Zender sets this sonnet in his 1999 work for soprano and chamber orchestra, Music to Hear. As Theodore Banks writes, “The first thirteen lines elaborate a comparison between the friend and music. Himself the embodiment of music, he does not want to hear it although he really loves it.” (Theodore Banks, “Shakespeare’s Sonnet No. 8,” *Modern Language Notes* 63/8 (1948), 541–542.) The sonnet is considered one of Shakespeare’s “procreation sonnets,” urging the young addressee to marry and have children.

EXAMPLE 7: Annotated score, *Bardo,* mm. 1–21
As above, numbers refer to the position of each pitch in the local harmonic series (indicated by pitch-class letter names). Arrows indicate the reinterpretation of a "pivot" pitch as a different harmonic over a new fundamental. (Note that the cello part is written for performance with a customized curved bow, which allows otherwise impossible multiple stops.)

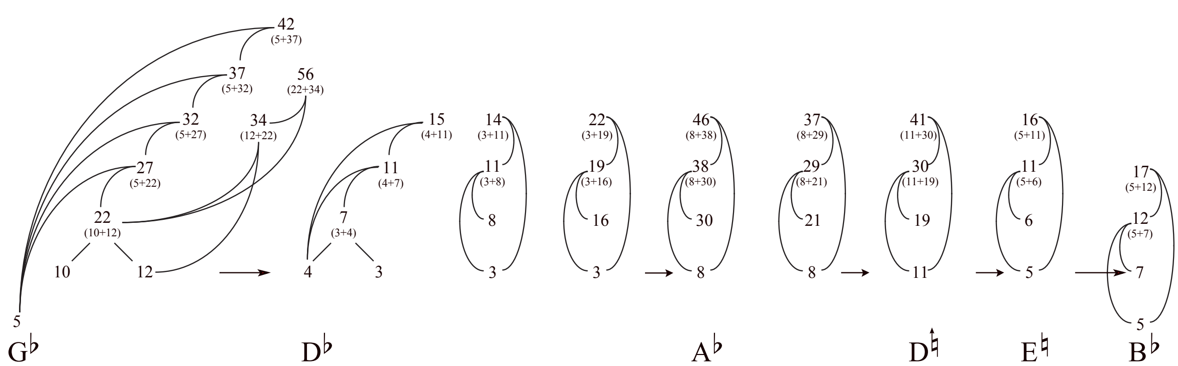


EXAMPLE 8: Genealogy of pitches in Example 7

This genealogy summarizes the passage analyzed in Example 7. As in the annotated score, numbers indicate the position of each pitch in the local harmonic series, and arrows show the reinterpretation of pitches over new fundamentals.

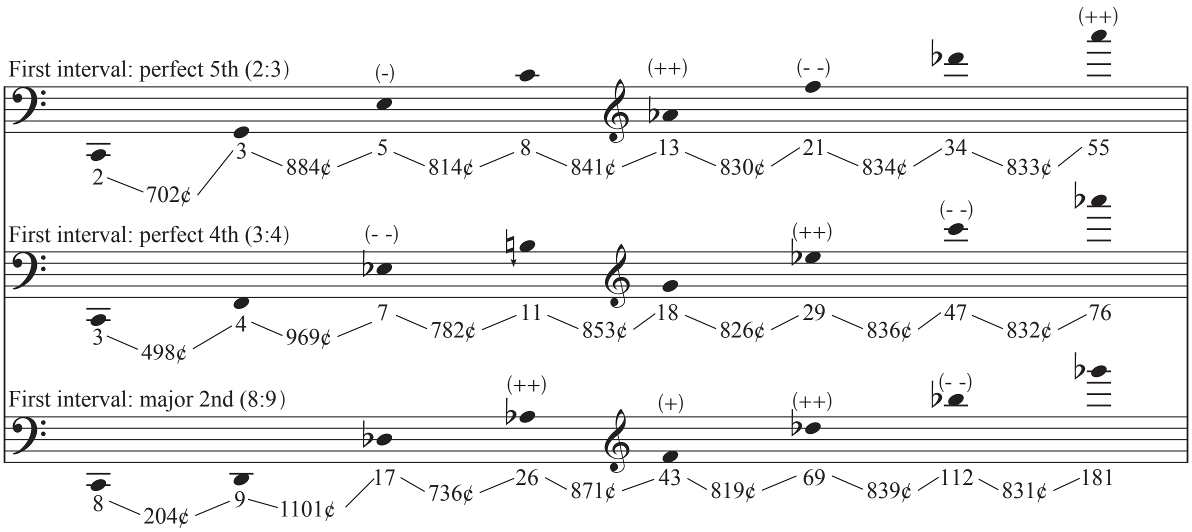
EXAMPLE 9: Pitch synopsis of *Music to Hear,* mm. 175–179

Like the excerpt from *Bardo* in Example 7, this passage is based on genealogies which expand simple pitch material into more complex formations: here, the starting points are the octave B♭s and the held D♭ that follows (overtones 5, 10, and 12 of a G♭ fundamental).



EXAMPLE 10: Genealogy of pitches in Example 9

This example charts the "descent" of each pitch from its forebears; the last seven chords share an identical genealogical graph, but different input notes result in varied pitch formations.



EXAMPLE 11: Fibonacci-type series of summation tones
Each new pitch is the summation tone of the two preceding pitches. Note the convergence of interval size toward 833 cents, the interval of the “golden ratio.”

EXAMPLE 12: *Bardo,* mm. 29–45 (cello solo only)
A musical application of the Fibonacci-type series of summation tones: here, the starting pitches C and C# (overtones 16 and 17 of a C fundamental) give rise to the series 16, 17, 33, 50, 83, 133, 216, 349. At measure 41, the 349th overtone is reinterpreted as the 22nd overtone of the sustained C_2.

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