Contemporary Music Review
Vol. 25, No. 3, June 2006, pp. 263–281

Routledge
Taylor & Francis Group

Tone Representation and Just Intervals in Contemporary Music

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Simple integer ratios between frequencies (just intervals) have historically been invoked as the basis of musical harmony. Hugo Riemann's theory of Tonvorstellung *proposes that we mentally 'represent' pitches as members of justly tuned triads. Broadening the scope of Riemann's theory to include more complex interval ratios, James Tenney argues that we understand heard intervals as approximations, within a tolerance range, of referential just intervals. This essay develops the idea of tone representation as an analytical tool for contemporary music. An excerpt from György Ligeti's* Melodien *(1971) is analyzed as a succession of just intervals pivoting above a shifting fundamental. A close reading of the opening of Gérard Grisey's* Vortex Temporum *(1994–1996) explores tone representations of complex microtonal harmonies, and examines the gap between compositional techniques and aural experience in this 'spectral' work.*

Keywords: Just Intonation; Hugo Riemann; James Tenney; György Ligeti; Gérard Grisey; Spectral Music

I

Throughout the history of music theory, just intervals have been defined by simple integer ratios—measured by ancient Greek theorists as ratios between string lengths on a monochord, and later (beginning in the sixteenth century) as ratios between vibration frequencies. Simple ratios such as 2:3 (the fifth) or 5:6 (the minor third) produce the pure intervals which are the basic building blocks of tonal music. However, much of the theoretical and analytical writing on twentieth-century music has focused on a different concept of interval: interval as the distance between two pitches measured in equal-temperament semitones. The important distinction between ratio and distance models of interval has been discussed in detail by Ben Johnston (1964). Reviving the idea of intervals as frequency ratios can help illuminate our experience of intervals as qualitative sonic phenomena, describing the particular acoustic qualities of each interval which are key to our understanding of their musical

ISSN 0749-4467 (print)/ISSN 1477-2256 (online)
DOI: 10.1080/07494460600726529

role. This ratio-based approach to interval can be useful for analyzing harmony in many contemporary musical works.

All just intervals can be expressed as integer ratios between the frequencies of two pitches. For example, the interval of the just major third C4/E4 can be expressed as the frequency ratio 4:5—we could also say that these pitches are in the same relationship as the fourth and fifth overtones of a low fundamental C2. Just interval ratios can be converted logarithmically into distance intervals measured in cents (hundredths of an equal-temperament semitone); the interval 4:5 is equivalent to a distance of approximately 386 cents, or 3.86 semitones.

Renaissance theorists such as Gioseffo Zarlino defined musical consonances as the intervals with ratios made only with the integers one through six and eight; these ratios describe the octave, fifth, fourth, and major and minor thirds and sixths in just intonation (along with their octave equivalents, such as the twelfth and double octave). By admitting higher multiples of two, three and five, one can create the just intervals which have historically been considered dissonances, such as the whole tone (8:9) or the semitone (15:16). The inclusion of higher prime numbers (such as 7, 11 and 13) and their multiples leads to intervals which fall between the customary semitone divisions of the octave, such as the flat natural seventh (4:7, or 969 cents) or the flat tritone (8:11, or 551 cents)—this is what Ben Johnston calls 'extended just intonation'.

Some twentieth-century composers have turned to these microtonal just intervals as the basis for new musical systems: extended just intonation has been explored by Harry Partch and Ben Johnston, and the spectralist composers Gérard Grisey and Tristan Murail have used various types of overtone-based harmonies. The interest in just interval and the closely related phenomenon of overtones is tied to a renewed interest in harmony and 'sound as sound', in reaction to the abstract distance-based conception of interval in serialism and motivic atonality. Adopting just interval as an analytical concept can provide a way to discuss this qualitative aspect of musical experience.

My aim is not to disparage the concept of interval as distance—its enormous power and practicality of application are undeniable. Rather, I propose that having recognized that interval can be understood in different ways, we may find applications where the less familiar concept of intervals as frequency ratios offers new analytical insights which would not come to light with a distance-based intervallic approach.

To this end, I will discuss excerpts from two major works from the last half of the twentieth century—György Ligeti's *Melodien* (1971) and Gérard Grisey's *Vortex Temporum* (1994 – 1996). In Ligeti's work, we shall see how the pitches of a chord progression can be understood as approximations, to the nearest semitone, of the high harmonics of a shifting fundamental tone, or 'root'. In Grisey's *Vortex Temporum,* we will explore the ambiguity which arises from his use of various distortions of an overtone series, and how Grisey's compositional techniques are often at odds with the aural sense we make of the complex resulting harmonies.

II

If just intervals are significant to our musical understanding, how do they apply to music written in equal temperaments, such as the Ligeti and Grisey works? We must accept that equal-temperament approximations of just intervals still carry some of the same musical qualities as their just intonation counterparts—in other words, that we can recognize the harmonic implications of just intervals even when they are not perfectly tuned. The relationship between just intervals and their mistuned counterparts has been addressed in compatible ways by two very different music theorists: the early-twentieth-century German theorist Hugo Riemann and the contemporary American composer-theorist James Tenney.

Riemann

In his idealist approach to music theory, Riemann asserts that we are not merely passive recipients of musical sounds, but active interpreters of those sounds into logical structures; the mental representation of musical relationships is more important than their actual manifestation as sound. Every musical tone is 'represented' or 'imagined' as part of a justly tuned major or minor triad. (Riemann's work focuses on the primarily triadic music of the tonal tradition.) The harmonic meaning of each tone depends on its context—whether it is, for example, the third of a minor triad or the fifth of a major triad. A single, isolated tone may pose problems of ambiguity—but once we are familiar with the piece of music in which it occurs, it takes on a character depending on its harmonic representation: 'According to whether a note is imagined as 1, 3, or 5 of a major chord or as I, III, or V of a minor chord, it is something essentially different and has an entirely different expressive value' (Riemann, 1992, p. 86).

Riemann labels the members of a major triad with the arabic numerals 1, 3 and 5, and the members of a minor triad with the roman numerals I, III and V. In a minor triad, intervals are labeled from the fifth of the chord downward rather than from the root upward—this is in keeping with Riemann's dualist conception of the minor triad as an upside-down version of the major triad. All of these triadic relationships are governed by just intonation: thus, the equal-tempered fifths and thirds of chords played on a piano are 'imagined' as pure, just intonation intervals in the listener's mind. Heard intervals are transformed into just intervals in the mind through the process of *Tonvorstellung*, which I will translate here as 'tone representation'. Tone representation also provides a harmonic meaning for each pitch, associating it with its harmonic root by a just interval.

Riemann proposes a general principle governing the way that our minds understand tones harmonically—we prefer the simplest possible interpretation consistent with the music. 'This *Principle of the Greatest Possible Economy for the Musical Imagination* moves directly toward the rejection of more complicated structures, where other meanings suggest themselves that weigh less heavily on the powers of interpretation' (ibid., p. 88 [emphasis in original]). Thus, given a collection of pitches, we will understand them as connected by the simplest possible just ratios, even when our ears are confronted by the complex and irrational intervals of equal temperament: 'our organ of hearing fortunately is so disposed that absolutely pure intonation is definitely not a matter of necessity for it' (ibid., p. 99).

Tenney

For Riemann, the possibilities of tone representation end with the tonal relationships found within a triad: just intonation thirds and sixths, fourths and fifths. James Tenney has proposed some similar theoretical ideas, but from a vastly different aesthetic position—while Riemann was aesthetically conservative, Tenney is a major creative figure in American experimental music. Like Ben Johnston and Harry Partch, he expands the concept of just interval to allow more complex integer ratios (Gilmore, 1995). However, unlike these more strict just-intonation advocates, Tenney allows what he calls *tolerance*—'the idea that there is a certain finite region around a point on the pitch height axis within which some slight mistuning is possible without altering the harmonic identity of an interval' (Tenney, 1992, p. 109). This notion brings Tenney very close to Riemann's idea of tone representation, but with a much more liberal attitude towards which ratios might act as 'referential' just relationships.

> Now, I propose as a general hypothesis in this regard that the auditory system would tend to interpret any given interval as thus 'representing'—or being a variant of—*the simplest interval within the tolerance range* around the interval actually heard (where 'simplest interval' means the interval defined by a frequency ratio requiring the smallest integers). The simpler just ratios thus become 'referential' for the auditory system...
>
> Another hypothesis might be added here, which seems to follow from the first one, and may help to clarify it; within the tolerance range, a mistuned interval will still carry *the same harmonic sense* as the accurately tuned interval does, although its timbral quality will be different—less 'clear', or 'transparent', for example, or more 'harsh', 'tense', or 'unstable', etc. (Tenney, 1992, p. 110 [emphasis in original])

Tenney separates the timbral quality of a heard interval from its harmonic sense; thus, we can imagine two different heard intervals representing the same harmonic sense, or the same heard interval representing two different harmonic senses. As an example, an equal-temperament major third (400 cents) played on a piano can represent a just 4:5 major third of 386 cents. The equal-tempered third is audibly sharper than the smaller just third (it has a different 'timbral quality'), but we can still identify it as projecting the 'harmonic sense' of the just interval. However, one might object that there is a just ratio that much more closely approximates the equal-temperament third: the 19:24 interval of 404 cents. Why then should we tend to understand the 400 cent equal-temperament third as 4:5, not 19:24? Tenney's explanation is in the same spirit as Riemann's 'principle of the greatest possible economy': 'Given a set of pitches, we will interpret them in the simplest way possible' (Tenney & Belet, 1987, p. 462). The context in which we hear an interval is thus very important in determining its harmonic sense. If we hear the equal-tempered third C/E as part of a larger set of pitches implying a fundamental of A, the context could lead us to understand it as the complex just ratio A(19:24). If we hear the same third on its own, though, we are more likely to understand it as the simpler ratio C(4:5).

Like Riemann, Tenney proposes that our mental representation of harmony is based on simple frequency ratios, and that we tend to experience slight deviations from a referential just interval as still representing that interval. This is not to say that we do not hear the specific sonic quality of, for example, an equal-temperament third as compared to a just third—we can notice the difference in intonation while still recognizing the harmonic sense of the referential just third.

Tenney does not specifically define what the tolerance range of a just interval might be, but he notes that the degree of tolerance tends to 'vary inversely with the ratio complexity of the interval' (Tenney, 1992, p. 110); that is, simple intervals such as octaves and fifths are more likely to be recognized in spite of mistunings, while complex relationships, such as the 19:24 major third, are likely to lose their identity if mistuned by a comparable amount.

Tools and Guidelines

Before exploring the applicability of tone representation in specific works, it will be useful to establish some basic theoretical tools. Tone representation is based on just intervals, which can be understood as intervals between the partials of an overtone series. Figure 1 shows the first to thirty-second partials of C1 rounded off to quartertones, with the distance above the fundamental in cents (mod 1200) of each pitch. (Quartertone rounding has been chosen here to match the quartertone notation of Grisey's *Vortex Temporum*; in other analytical contexts, it may be more practical to use coarser or finer approximations. For example, my analysis of Ligeti's *Melodien* rounds each partial to the nearest semitone.)

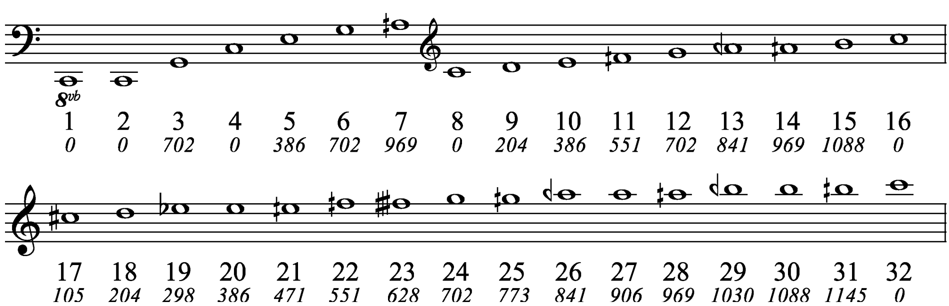

Figure 1 Overtone series on C, partials 1 to 32.

For the current study, I will limit tone representation to the just intonation intervals which can be formed using the integers 1 to 21 and their products when multiplied by powers of 2—that is, the partials 1 to 21 and their octave equivalents. By analogy to the concept of pitch class, the collection of an integer and its products when multiplied by powers of 2 could be thought of as a *partial class*. Thus, we will admit into our possibilities of tone representation the intervals 13:17 and 17:26 (26 is twice 13, which falls within our partial class limit of 21), but not 17:25. Using partial classes ensures that the inversions and octave equivalents of every interval within the partial class limit of 21 will also fall within that limit; so, in addition to the 15:17 major second of 217 cents, we will include its inversion to a 17:30 minor seventh and its octave equivalent 15:34 major ninth. The idea of partial class is derived from Riemann's own formulation of octave equivalence for triadic tone representation:

> Through the equivalence, or at least the more intimate relationship, of *tones* that *stand* in *octave relation,* the number of tones belonging to a single harmony is reduced to three altogether: the *prime* (and its octaves = 1:2:4:8:16, etc.), the *fifth* (and its octaves = 3:6:12, etc.), and the *third* (and its octaves = 5:10:20, etc.), above as well as below. (Riemann, 1992, p. 98 [emphasis in original])

Why limit tone representation to partial classes of 21 and below? In examining music which uses quartertone approximations, such as the Grisey work analyzed below, errors in rounding begin to become a serious problem with the inclusion of higher partials. Furthermore, with the lower degree of tolerance of the complex intervals involving partial classes above 21, it is difficult to convey a strong sense of their identity when they are approximated to a quartertone grid.

A basic notational convention will make the description of tone representations much simpler. To indicate the tone representation 'the fourth and fifth partials of an F fundamental', I will write F(4:5). The same notation is applicable to single pitches, e.g. B(17), or to larger collections, e.g. A(11:13:17:21:24).

Figure 2 uses this notation to list all of the intervals (within an octave) which use the partial classes 1 to 21. They are sorted by their quartertone approximations (cf. Figure 1); the symmetrical layout of the chart puts each interval opposite its inverse modulo 12 (for example, 1½ is opposite 10½). An instance of each interval is shown beginning on the pitch class C. The possible representations of that interval are then listed: for example, the tritone C/F-sharp can be either B(17:24) or F(12:17). These tone representations are sorted by the size of the just interval, which is shown (in cents) beneath each ratio. The rounding off of each partial to the nearest quartertone leads to some anomalies between an interval's quartertone notation and its just intonation size: for example, the interval 20:21 (84 cents) is rounded down to a quartertone (50 cents), even though the smaller interval 21:22 (81 cents) is rounded up to a semitone (100 cents). If the existence of these rounding anomalies is kept in mind, they should not present a major obstacle when looking up possible tone representations for a given interval.

6 C—F♯
B(17:24) F(12:17)
597 603

5½ C—F𝄲
D𝄲(14:19) G𝄳(11:15) A(19:26) C(8:11)
529 537 543 551
E𝄳(13:18) G♯(5:7) C♯(15:21)
563 583 583

5 C—F
F(3:4) B♭(9:12) C♯(15:20) G𝄲(21:28)
498 498 498 498

4½ C—E𝄲
D𝄲(7:9) B(17:22) G♯(10:13) E𝄳(13:17)
435 446 454 464
C(16:21)
471

4 C—E
G𝄲(21:26) C(4:5) F(12:15) A(19:24)
370 386 386 404
C♯(15:19) G𝄳(11:14)
409 418

3½ C—E𝄳
D𝄲(14:17) B♭(9:11) E𝄳(13:16) B(17:21)
336 347 359 366

3 C—E♭
B(17:20) G𝄳(11:13) C(16:19) G♯(5:6)
281 289 298 316
C♯(15:18)
316

2½ C—D𝄲
D𝄲(7:8) G𝄲(21:24) E𝄳(13:15) A(19:22)
231 231 248 254
F(6:7) B♭(18:21)
267 267

2 C—D
B♭(9:10) B(17:19) C(8:9) C♯(15:17)
182 193 204 217

1½ C—D𝄳
D𝄲(14:15) F(12:13) G𝄳(11:12) G♯(10:11)
119 139 151 165
A(19:21)
173

1 C—C♯
G𝄲(21:22) A(19:20) B♭(18:19) B(17:18)
81 89 94 99
C(16:17) C♯(15:16) E𝄳(13:14)
105 112 128

½ C—C𝄲
G♯(20:21)
84

6½ C—G𝄳
D𝄲(7:10) G𝄲(21:30) B♭(9:13) G𝄳(11:16)
617 617 637 649
E𝄳(13:19) C♯(15:22) A(19:28)
657 663 671

7 C—G
C(2:3) F(6:9) G♯(10:15) D𝄲(14:21)
702 702 702 702

7½ C—G𝄲
G𝄲(21:32) B(17:26) E𝄳(13:20) G𝄳(11:17)
729 736 746 754
B♭(9:14)
765

8 C—G♯
D𝄲(7:11) A(19:30) F(12:19) G♯(5:8)
782 791 796 814
C♯(15:24) E𝄳(13:21)
814 830

8½ C—A𝄳
G𝄲(21:34) C(8:13) G𝄳(11:18) B(17:28)
834 841 853 864

9 C—A
F(3:5) B♭(9:15) A(19:32) E𝄳(13:22)
884 884 902 911
G♯(10:17)
919

9½ C—A𝄲
D𝄲(7:12) G𝄲(21:36) G𝄳(11:19) C♯(15:26)
933 933 946 952
C(4:7) F(12:21)
969 969

10 C—B♭
B(17:30) B♭(9:16) A(19:34) G♯(5:9)
983 996 1007 1018

10½ C—B𝄳
G𝄲(21:38) G𝄳(11:20) F(6:11) E𝄳(13:24)
1027 1035 1049 1061
C♯(15:28)
1081

11 C—B
D𝄲(7:13) C(8:15) B(17:32) B♭(9:17)
1072 1088 1095 1101
A(19:36) G♯(10:19) G𝄳(11:21)
1106 1111 1119

11½ C—B𝄲
G𝄲(21:40)
1116

Figure 2 Chart of just intervals using the partial classes 1 to 21.

This chart will be useful in determining possible tone representations for a collection of pitches. We can define several guidelines for choosing a tone representation for a given collection:

1) Use the smallest possible number of fundamentals; invoke multiple fundamentals only if they yield a significantly simpler solution than a single fundamental can. Collections of pitches which can be related to the same fundamental will tend to imply that fundamental as a kind of 'root'—especially when the partial class of the fundamental (1, 2, 4, 8, etc.) is present (see Parncutt, 1988 and Väisälä, 2002). As Ben Johnston observes:

   > A group of pitches may be very complexly related to one another, but often all of them can be simply related to another pitch, which need not even be present. Thus, the missing pitch is strongly *implied* by the complex group. The *root* of a chord, the *tonic* of a tonality, the *principal tonality* of a modulating movement are all examples of this principle. (Johnston, 1964, p. 61 [emphasis in original])

2) Use the 'most economical' possible representation of a pitch collection—that is, choose the representation which uses the smallest integers in its interval ratios. A corollary of this is that intervals such as fourths and fifths, which can be represented by low integer ratios, should use these low integers when possible—this means that they will have a stronger effect on the determination of a collection's fundamental than more complex intervals. Paul Hindemith, who also argues for a version of *Tonvorstellung* ('our ability to accept complex intervals as versions of their nearest simple equivalents'), makes a similar point when he states that the root of a chord is determined by its 'best' interval—the interval with the simplest just intonation ratio (Hindemith, 1942, pp. 94–98).
3) Prefer interpretations in which the just intervals of the tone representations correspond closely to the actual intonation of the music—that is, interpretations which require the least possible mental retuning from heard intervals to the referential just intervals.
4) Consider the position of pitches in a chord: the lower a pitch sounds, the more weight it should be given when considering harmonic relationships.

III

Ligeti's *Melodien* for orchestra is not a microtonal piece (it's written entirely in standard twelve-tone notation), but certain passages strongly imply tone representations which involve microtonal intervals such as 7:8 (231 cents) and 8:11 (551 cents). These just microtonal intervals are approximated to the nearest semitone—in a sense, then, one could argue that these passages are examples of microtonal music forced into a semitone grid. Ligeti has long been interested in microtones and overtones; Richard Toop (1999) discusses an abandoned electronic piece from the late 1950s based on a 'synthetically produced overtone series' of sine tones. Microtones produced as high string harmonics appear in the first movement of the Cello Concerto (1966) and the Second String Quartet (1968); the Double Concerto (1972), Ligeti's next work after *Melodien*, incorporates notated microtones in the first movement. Bob Gilmore (2003, pp. 27 – 30) discusses how Ligeti's 1972 encounter with Harry Partch and his music influenced the Double Concerto and later works. Ligeti's continuing interest in microtones and just intonation is evident in such recent works as the Violin Concerto (1990 – 1992), the Viola Sonata (1994) and the Hamburg Concerto (1998 – 1999).

Figure 3 is an abstraction of all the pitches from m. 11 to m. 19 of *Melodien* (an excerpt of about 40 seconds). Jonathan Bernard (1999, pp. 3 – 10) has discussed part of this section in terms of transformations in pitch space (that is, the space of pitches in register, as opposed to modular pitch class space). In this approach, interval is conceptualized as distance; Bernard's analytical diagram of the music as a 'graph of durations in pitch space' makes the analogy of interval to spatial distance explicit.

Turning from distance intervals to just intervals, we can arrive at a very different reading of the passage as an example of changing tone representations. The excerpt begins with a unison A6. Given the lack of a context which would imply otherwise, it seems reasonable to assign this pitch the simplest possible tone representation of 'fundamental': A(1).

At m. 13, an F appears below the A (this dyad is doubled an octave above by the celesta, but this does not significantly affect our harmonic understanding). The simplest, most 'economical' way to hear the F/A dyad is as a just major third, F(4:5). At this moment, the tone representation of the A changes—it changes from a fundamental (1) to a just major third (5) above the new fundamental F.

The addition of E-flat in m. 14 does not affect our sense of F as root—rather, it sounds like the approximated seventh harmonic of F(7:8:10). Other tone representations of these three pitches are possible, such as E-flat(8:9:11), B(10:11:14), G(13:14:18), or D(17:19:24), but with no reason to favor these more complex representations, the more economical F(7:8:10) is a better choice. The F has also been already established as a fundamental in m. 13, so a sort of 'inertia' makes us likely to keep the same partial classes for the F and A. In music which rounds off microtonal just intervals to a semitone grid, it is necessary to accept larger degrees of tolerance

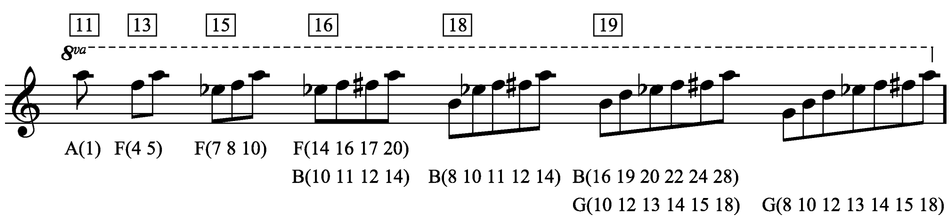

Figure 3 Pitch collections in *Melodien*, mm. 11 – 19.

than in music rounded to a quartertone grid—thus, the 200-cent interval E-flat/F is taken to represent 7:8 (231 cents) here, with a 31-cent discrepancy between the heard and referential intervals. While F and A are still represented by the same partial classes as in m. 13, the representation of the sonority must be understood as occurring an octave higher in the harmonic series (changing the representation of F from 4 to 8), to allow the inclusion of E-flat as F(7). The move into higher integers corresponds with the addition of more complex and less traditionally consonant intervals.

At m. 16, the addition of an F-sharp creates a harmonically ambivalent situation. We can fit the new collection into an F spectrum, F(14:16:17:20), which requires a move still higher into the overtone series, reinterpreting F from 8 to 16. However, a competing interpretation emerges which better satisfies the 'Law of Economy': we can hear the collection as an approximation of B(10:11:12:14). It's difficult to choose between the two; our sense of the F fundamental is weakened, but the sense of B as fundamental is still not established (particularly as the pitch class B is not part of the collection). The addition of a B to the sonority in m. 18 resolves this ambiguity, and decisively shifts our sense of fundamental from F to B. The fifth B/F-sharp (one of the simplest just intervals) plays a strong role in confirming B as the root. So, now our *Tonvorstellung* of the total sonority is B(8:10:11:12:14).

As our sense of fundamental changes from F to B, the harmonic sense we make of each of the pitches also changes: for example, the E-flat stops sounding like a natural seventh above the fundamental, or F(7), and takes on the quality of a major third: B(10). The tone representation of the interval F/A changes from F(8:10), at 386 cents, to B(11:14), at 418 cents. Our tolerance for mistuning of these just intervals makes it possible to hear the equal-tempered third of 400 cents as an approximation of either interval, and as a pivot between the two fundamentals. Thus, by accepting a degree of tolerance in our tone representations, complex just intonation patterns can be conveyed in twelve-tone equal temperament.

The added D in m. 19 throws our recognition of B harmony into doubt, much as the F-sharp weakened our sense of the F fundamental in m. 16. We can persist with a B tone representation of B(16:19:20:22:24:28), but the simpler option G(10:12:13:14:15:18) is more aurally convincing—and, analogously to the arrival of B in m. 18, the G eventually appears a fifth below the D to confirm this reading. Note that the shift in fundamentals from B to G mimics the shift from A to F at the beginning of this passage.

In the bars following m. 19, Ligeti continues to add pitches more rapidly; our analysis can keep up with only a few more additions before the density of pitches overwhelms our capacity to discern a clear harmonic structure. After this point, a motivic or transformational analysis (such as Bernard's) could better describe the music's progress. The tone representations discussed here can easily coexist with such atonal, piece-specific approaches to analysis. Motivic and atonal interval structures may also take part in purely harmonic processes, which tone representation can describe in detail.

Unlike motivic or transformational analyses, which focus on unique *intraopus* relationships such as the repetition or transformation of characteristic motives or pitch collections, this analysis has used tone representation to describe how the unfolding of pitches creates a sense of shifting just intonation sonorities. In place of unique, piece-specific relationships, the set of just intervals is taken as an *interopus* constant governing our harmonic perception. That is, instead of assuming a completely atonal world, in which the only landmarks are motivic correspondences within a work, an analysis using tone representation posits a world of extended tonality governed by a consistent set of just harmonic relationships.

IV

Vortex Temporum, a three-movement work for flute, clarinet, violin, viola, cello and piano, poses some intriguing analytical difficulties. The work is a 'spectral' composition, insofar as it is composed by reference to models based on the acoustic spectra of instruments. However, Grisey's compositional techniques significantly alter these spectra, sometimes to the point of unrecognizability.

The compositional techniques and plans which Grisey used to construct the work are described in detail in studies by Jérôme Baillet (2000) and Jean-Luc Hervé (2001)—and substantial corroborating sketches are also held by the Paul Sacher Stiftung. The description of compositional process, however, is not necessarily a good description of a piece's aural and musical effect. Even though many spectral techniques take acoustic and psychoacoustic facts as their starting point, there is often no clear, unambiguous relationship between such compositional techniques and their audible musical results.

Manfred Stahnke (2000) has made some brief but tantalizing observations about how some of the harmonies of *Vortex Temporum* might be interpreted by ear—his interpretations address aspects of the harmonies which are not evident from a consideration of their compositional origin. By examining the beginning of the work's first movement through the lens of tone representation, I hope to offer some new insights into its harmonic relationships as I hear them. Tone representation can function as a sort of 'listening grammar' for complex microtonal sonorities.

The first two minutes of the first movement use a very limited set of harmonic materials—we hear the alternation of three distinct 'chords', arpeggiated by the flute, clarinet and piano. Each of these three chords was conceived by Grisey as a subset of a 'stretched' harmonic series. The normal harmonic spectrum is systematically distorted, so that each octave is stretched to approximately an octave plus a quartertone. From the resulting distorted spectra, Grisey selects certain pitches for each chord. Figure 4, drawing on Baillet and Hervé, illustrates the derivation of the three chords (labeled, in order of appearance, *x*, *y* and *z*).

The stretched harmonic spectrum is conceived by analogy to the mildly inharmonic spectra of many common musical sounds: the upper harmonics of a piano string, for example, are slightly sharp in comparison to the precise multiples of

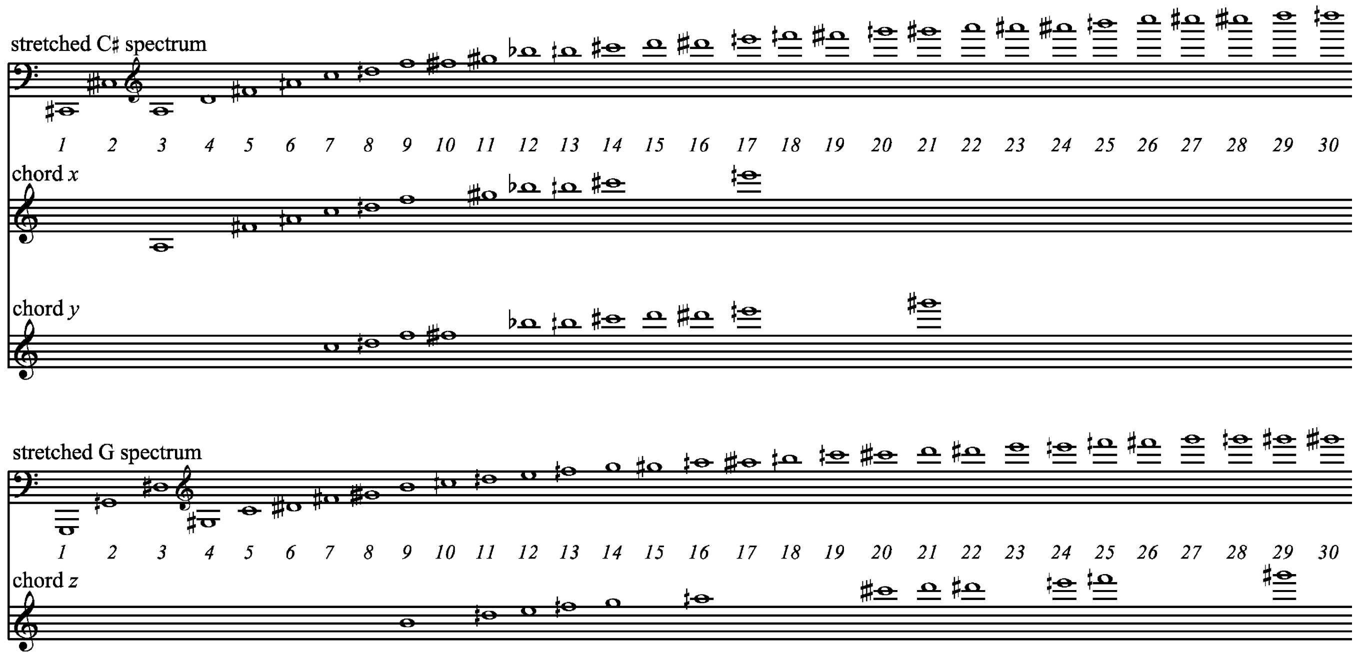

Figure 4 Derivation of *Vortex Temporum* chords *x*, *y* and *z* from stretched spectra.

the fundamental frequency. If Grisey's more intensely stretched spectra were played in their entirety, and with simple sine tone partials, we might recognize their connection to this acoustic model. However, in *Vortex Temporum*, only a small subset of each spectrum is heard, and that is 'resynthesized' using complex instrumental timbres. The selection of a subset of pitches from a spectrum can completely efface the spectral derivation of that subset—careful selection of pitches can even imply that their source is a different spectrum altogether. Grisey's own awareness of this possibility is clear from a sketch in the Sacher Stiftung—by selecting carefully from the stretched G spectrum in Figure 4, he derives subsets which imply a stretched D-sharp spectrum (D-sharp/D-¾sharp/B/E/G-sharp—partials 3, 6, 9, 12 and 15) and even a compressed F-sharp spectrum (F-sharp/F-¼sharp/C-¼sharp/F-¼sharp—partials 7, 13, 19 and 25).

Given this gap between spectral compositional procedures and the aural effect of the derived chords, recounting the compositional process sheds little light on the way the harmonies are actually heard. It will be more productive to analyze the chords without reference to their derivation, concentrating instead on how we hear the chords: their tone representations, internal tensions, and relationships to one another.

Figure 5 shows how the three chords *x, y* and *z* are deployed in rehearsal numbers 1 through 19 (this figure is based loosely on Baillet, 2000, p. 214). At each rehearsal number, the arpeggiation is punctuated by a cluster of very high piano notes, and sometimes quick notes in the strings—however, these short-lived punctuations do not seem substantially to affect the harmonic perception of the three sustained arpeggio chords, so they will be omitted from this analysis. Sometimes, the arpeggiation is joined by sustained single tones in one of the strings (shown here as half notes followed by bars indicating their length). As fixed points opposing the rapidly moving arpeggios, these sustained tones capture our aural attention very strongly, and they will be given corresponding weight in our analysis.

Chord x

Figure 6 illustrates some possible tone representations for the chords *x, y* and *z.* (Boxed pitches indicate the notes which are sometimes held as 'pedal points' by the strings.) At the beginning of *Vortex Temporum*, the arpeggiation of chord *x* is sustained for more than thirty seconds; it seems false to musical experience to assert that we don't begin making sense of this chord until there's something to compare it to. One advantage of tone representation is that we can say things about collections of pitches without comparing them to other collections, as in pitch-class set analysis. The technique of pitch-class set analysis is essentially comparative; in technical terms, this means that we can say very little about a pitch collection until we find something to compare it to. In many musical situations, however, we will want to say something right away. The idea of tone representation will allow us to make observations about the internal composition of this complex chord from the very

Figure 5 Arrangement of chords *x*, *y* and *z* in *Vortex Temporum*, rehearsal numbers 1 to 19.

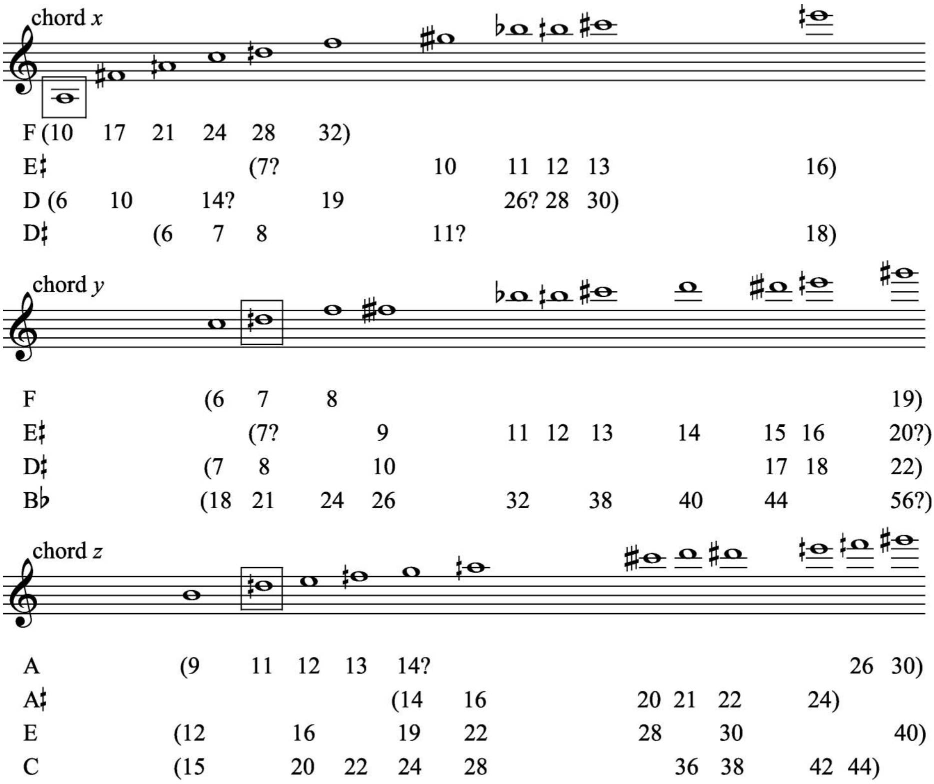

Figure 6 Tone representation analysis of *Vortex Temporum* chords *x*, *y* and *z*.

beginning of the work. This analytical approach does not view the identification of motivic repetition within a work as the most significant type of relationship—instead, we can cite the harmonic tendencies of any given collection, along with any internal tensions or ambiguities of tone representation. These tensions are essential in making the chord compelling to listen to for such a long time.

One way of hearing the first chord is as a combination of tones related to two harmonic centers, D and D-¼sharp (see Figure 6): the A and F-sharp imply a D root, while the A-¼sharp and D-¼sharp imply D-¼sharp. This tone representation is essentially what we would expect from the compositional derivation of the chords: the stretching process moves the C-sharp 'fundamental' up to D, then to D-¼sharp. However, the pitches G-¾sharp and B-flat don't fit well with either the D or D-¼sharp fundamentals (the question marks on Figure 6 indicate that the corresponding pitches are unusually out of tune for the indicated tone representation). Describing the chord as based on D or D-¼sharp (or, for that matter, as a stretched C-sharp spectrum) gives only an overall, statistical impression, which doesn't account well for details; to me, though, the harmonic details are precisely what make this chord interesting.

Another, more nuanced interpretation recognizes an F root for the lower part of the sonority, combined with an E-¼sharp root for the upper part. An advantage of this interpretation over the D/D-¼sharp one is that we can recognize a definite tone representation for every pitch, accepting less mistuning between the sounding pitches and their just intonation tone representations. Aurally, I find this tone representation much more convincing, even though one must accept quite high partial classes, 17 and 21, to account for the F-sharp and A-¼sharp as part of the F spectrum. A low F played beneath the harmony seems to fit well as a 'root' for the lower half of the chord; this is largely due to the harmonic strength of the fourth C/F (an interval which was problematic in the D/D-¼sharp reading). This interpretation gives us a clear explanation for the prominent C-F fourth, and does not require that we distort it into a more complex interval.

The arpeggio figuration tends to temporally separate the upper and lower parts of the chord. Playing the upper part of the chord alone (from the G-¾sharp up) makes its orientation towards E-¼sharp clearly audible. By recognizing a clearly delimited E-¼sharp collection above the F collection, the sonority seems much more like a *compressed* spectrum than a stretched one (as the chord's derivation would suggest); we also have a good explanation for the quartertone-flat octaves F/E-¼sharp and C/B-¼sharp, which are responsible for much of the chord's characteristic tension.

The ambiguity between the two possible tone representations of chord *x*—as a stretched spectrum with D and D-¼sharp roots or a compressed spectrum on F and E-¼sharp—seems to reflect an essential part of Grisey's musical style. As Stahnke (2000) writes, 'Grisey plays with the shape-finding capability (*Gestaltfindungsfähigkeit*) of our ear, with the thresholds of our awareness' (p. 383). The play between two plausible tone representations helps to animate the chord through this long passage.

Chord y

The first major harmonic change in the piece occurs at rehearsal number 6, with the move to chord *y*. As we know, this collection shares a common origin with chord *x*; both are subsets of the same stretched spectrum. We can easily hear that the two chords are closely related, since more than half of chord *y*'s pitches are common tones with chord *x*. Does this mean that the chords have the same harmonic effect? In fact, the changed pitches, and particularly the overall higher register, make the new chord's tone representations subtly different from those of chord *x*.

We hear a continuation of the F/E-¼sharp tone representation quite strongly—in fact, the F root of the lower part of the chord is much stronger than in chord *x*, in the absence of the F-sharp and A-¼sharp which were fairly weak partial classes of F. Without these pitches, the C/F fourth can be represented as 6:8 rather than the more complex 24:32. Three of the four new pitches (F-¾sharp, D, and D-¾sharp) fit well with the E-¼sharp fundamental, while the high G-sharp could be F(19) or a rather flat E-¼sharp(20).

The sense that the chord might be heard (at least in part) as based on D is considerably weakened by the absence of the low A and F-sharp. However, we can still sense the possibility of D-¼sharp as a root for three of the lower notes of the chord (C, D-¼sharp and F-¾sharp, but not F-natural). This is a byproduct of the stretched spectrum in the compositional process—if we take a sample from further up in the spectrum, the higher the perceived 'root' of that sample will be. The sense of D-¼sharp as fundamental is strengthened by the absence of the A and F-sharp, which were 'out of tune' with the D-¼sharp root, and by the added F-¾sharp, which is D-¼sharp(10).

It's also possible to recognize a weaker tone representation based on B-flat. The strength of the B-flat interpretation is that it can relate nearly all of the pitches to a single fundamental, but the very high partial numbers that it requires make it seem less convincing than the F/E-¼sharp interpretation.

When the music returns to chord *x* from chord *y* at rehearsal number 7, our perception of chord *x* is subtly colored by the experience of chord *y*. The sustained D-¼sharp, a sustained note in the viola in chord *y*, is heard as closely linked to the sustained cello A in chord *x*. The strength of this linkage makes an 'in tune' version of the interval A/D-¼sharp desirable—since this interval is clearly emerging as a harmonically important one, it makes sense to understand it as a representation of a just interval. Only the F tone representation allows us to hear the interval between A and D-¼sharp as a just relationship in the context of chord *x*, F(10:28), further strengthening the sense of an F root as opposed to D or D-¼sharp.

Chord z

At rehearsal number 10, we hear chord *z* for the first time. The pitches of this chord were selected from a spectral source set a tritone lower than that of chords *x* and *y*. Is there any way that we experience this harmonic change as a tritone transposition? Many of the pitches of chord *z* are a tritone below pitches in chords *x* and *y* (F, G-¾sharp, B-flat, B-¼sharp and C-sharp in chord *x* map onto B, D-¼sharp, E, F-¼sharp and G in chord *z*). However, the overall range remains the same, which works against the sense of tritone transposition; also, as Stahnke (2000) points out, the chords are linked by close voice leading, creating a sense of continuity with chord *x* and weakening the sense of tritone motion: 'In comparison to the spectrum on C-sharp, we experience an almost stationary (*gleichbleibendes*) tone field, that seems only to be illuminated by a different light. The quartertone movements are important for this effect' (p. 382).

Even acknowledging the many pitches in chord *z* which are related by a tritone to pitches in chord *x*, I don't hear an overall transposition by tritone here. To me, the most aurally convincing account of the sonority is as a combination of pitches implying an A root in the lower part of the chord and an A-¼sharp root in the upper part. (The sense that the sustained cello A of chord *x* lingers aurally into chord *z* strengthens this tone representation.) While in chords *x* and *y*, the upper part of the chord was based on a fundamental a quartertone below the fundamental of the lower part (E-¼sharp above F, or in spectral lingo, a 'compressed' spectrum), here, the upper part of the chord is based on a fundamental which is a quartertone *above* that of the lower part (A-¼sharp above A, or a 'stretched' spectrum). The contrast between these two types of tone representations further weakens the case for a tritone transposition—we have a sense of a different type of harmony, not a transposition of the same type. If we do hear a change of overall 'root' from chord *x* to chord *z*, it is likely to be from F to A—transposition up a third, not down a tritone.

It's also possible to hear tone representations of E and C in this sonority—particularly before the entrance at rehearsal number 16 of the sustained viola D-¼sharp, which contradicts both of these tone representations. The D-¼sharp strongly focuses our tone representations toward the A root for the lower notes of the chord; the lowest trichord, B/D-¼sharp/E, is only possible with A as fundamental: A(9:11:12). The common D-¼sharp sustained tone with chord *y* also makes chord *z* sound similar to *y* in some ways—both are heard as contrasts to the more ubiquitous chord *x*. The shared high G-sharp and nearly identical overall register also help to cement a relationship between the chords, although they are harmonically quite different.

The return of the D-¼sharp over chord *z* as a sustained tone is a striking harmonic event; this sustained pitch has previously only appeared with chord *y*, and we hear it quite differently in its new context. D-¼sharp is now heard as A(11) instead of F(7), and the dyad of sustained string notes A/D-¼sharp is reinterpreted to A(4:11) from F(10:28). The reinterpretation of this emphasized dyad, highlighted by the long duration of its component notes, is heard as an essential part of the harmonic move from F to A. (One can find a similar harmonic relationship, but in reverse, in *Melodien*: the dyad B/E-flat changes from B[16:22] to G[10:14].)

The close reading offered here indicates a possible route towards a deeper engagement with the harmonies of spectral music. By moving away from a description of compositional tools towards an analytical method which describes audible harmonies, we can begin to describe and discuss the fascinating tensions and ambiguities of this harmonic language in detail.

V

Tone representation is a highly flexible analytical tool—it can be applied to styles ranging from tonal music (as in Riemann's original formulation) to 'atonal' works in twelve-tone equal temperament such as *Melodien* or quartertone works such as *Vortex Temporum*. The idea of interval as ratio helps to reflect important aspects of musical experience which are not dealt with convincingly by distance-based approaches to interval—particularly the effect of specific intervallic qualities and the acoustic relationships of pitches and intervals to their fundamentals or roots. Tone representation provides an alternative way to explore our harmonic intuitions, and can shed new light on the ways that we come to understand the music we hear.

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